# Section 8.1 Quadratic Equations

Example 4 page 502

$$4x^2 + 9 = 0$$
$$4x^2 = -9$$
$$x^2 = -\frac{9}{4}$$
$$\sqrt{x^2} = \sqrt{-\frac{9}{4}}$$
$$x = \pm\frac{3}{2}i$$

Notice that we got 2 solutions!

Example $4\frac{1}{2}$

*A*

$$(x-3)^2 = 25$$
$$x^2 - 6x + 9 = 25$$
$$x^2 - 6x - 16 = 0$$
$$(x-8)(x+2) = 0$$
$$x = 8 \quad x = -2$$
$$x \in \{8, -2\}$$

*B*

$$(x-3)^2 = 25$$
$$\sqrt{(x-3)^2} = \sqrt{25}$$
$$x - 3 = \pm 5 \quad \bigstar$$
$$x = 3 \pm 5$$
$$x = 8 \quad x = -2$$
$$x \in \{8, -2\}$$

★ Remember this is a shortcut from $|x-3| = 5$

Example 5 page 502

$$(x-2)^2 = 7$$
$$\sqrt{(x-2)^2} = \pm\sqrt{7}$$
$$x - 2 = \pm\sqrt{7}$$
$$x = 2 \pm \sqrt{7}$$

Look at the check to be sure it was properly solved:

$$(x-2)^2 = 7$$
$$\left(2+\sqrt{7}-2\right)^2 = 7$$
$$\left(\sqrt{7}\right)^2 = 7$$
$$7 = 7$$

$$(x-2)^2 = 7$$
$$\left(2-\sqrt{7}-2\right)^2 = 7$$
$$\left(-\sqrt{7}\right)^2 = 7$$
$$7 = 7$$

Example 6 page 503

Solve:

$$x^2 + 6x + 8 = 1$$
$$(x + 4)(x + 2) = 1$$

But nobody cares that $x^2 + 6x + 8$ factors because the equation says it is equal to ONE (i.e. It needs to be equal to zero!)

What about:

$$x^2 + 6x + 9 = 2$$
$$x^2 + 6x + 7 = 0 \qquad \text{Nope. Doesn't factor!}$$
$$(x + \quad)(x + \quad) = 0$$

Notice $x^2 + 6x + 9 = (x + 3)^2$

$$x^2 + 6x + 9 = 2$$
$$(x + 3)^2 = 2$$
$$\sqrt{(x + 3)^2} = \pm\sqrt{2}$$
$$x + 3 = \pm\sqrt{2}$$
$$x = -3 \pm \sqrt{2}$$

This method worked because we had a perfect square on the left side.

If we do not have that situation, we can create it by using the method called

“**Completing the Square**”

Example 7

$$x^2 + 6x + 4 = 0$$
$$x^2 + 6x \quad = -4 \qquad \text{we move the constant to the right side}$$

We need the missing constant on the left. We want to end up with a perfect square. The quickest method for determining that constant is to take “half of the coefficient of “x-term” and squaring it. Half of 6 is 3. Squared is 9.
***We will add 9 to both sides of the equation to maintain equality.*** Thus:

$$x^2 + 6x + 9 = -4 + 9$$
$$(x + 3)^2 = 5$$
$$x + 3 = \pm\sqrt{5}$$
$$x = -3 \pm \sqrt{5}$$

Let’s check to show that these really work.

$-3+\sqrt{5}$ $\left(-3+\sqrt{5}\right)^2+6\left(-3+\sqrt{5}\right)+4=0$ in *original* equation.

$$9 - 6\sqrt{5} + 5 - 18 + 6\sqrt{5} + 4 = 0$$

$$0 = 0$$

The check for $-3-\sqrt{5}$ works in the same manner.

Example 9 page 505

$$x^2+5x-3=0$$

$$x^2+5x+\quad\;=3$$

$$x^2+5x+\left(\frac{5}{2}\right)^2=3+\frac{25}{4}$$

$$\left(x+\frac{5}{2}\right)^2=\frac{37}{4}$$

$$x+\frac{5}{2}=\frac{\pm\sqrt{37}}{2}$$

$$x=\frac{-5\pm\sqrt{37}}{2}$$

Example 10 page 506

$3x^2 + 7x - 2 = 0$ Notice the leading coefficient is a 3!

We ***must*** **have the leading coefficient a one**.

We multiply the entire line by one-third to get:

$$x^2 + \frac{7}{3}x - \frac{2}{3} = 0$$

$$x^2 + \frac{7}{3}x = \frac{2}{3}$$

$$x^2 + \frac{7}{3}x + \left(\frac{7}{6}\right)^2 = \frac{2}{3} + \frac{49}{36}$$

Notice $\frac{2}{3} = \frac{24}{36}$

$$\left(x + \frac{7}{6}\right)^2 = \frac{73}{36}$$

$$x + \frac{7}{6} = \pm\frac{\sqrt{73}}{6}$$

$$x = \frac{-7 \pm \sqrt{73}}{6}$$